Title: Patterns with Equilateral Triangles

Brief Overview:

Students will explore two different patterns created by equilateral triangles. They will use a geometric model to establish a data table and then develop a formula to describe the relationship between the variables. Students will use this formula to make predictions about answers that would be very difficult to find with a concrete model. Finally, students will use the graphing calculator to plot their data and determine an appropriate model that would fit the data (linear, quadratic, exponential).

Link to Standards:

- **Problem Solving** Students will demonstrate their ability to solve mathematical problems by analyzing concrete models to develop a formula for future predictions.
- **Communication** Students will discuss with each other and write their conclusions based on their interpretation of tables and graphs.
- **Reasoning** Students will demonstrate their ability to reason mathematically. They will make conjectures and support their arguments.
- **Connections** Students will demonstrate their ability to connect geometric models to algebraic equations and statistical analysis.
- **Number & Number Relationships** Students will demonstrate their ability to use estimation strategies to establish a formula to represent a sequence.
- **Patterns & Functions** Students will generalize a relation from a table and describe how a change in one variable results in a change in the other.
- **Algebra** Students will develop an algebraic formula to represent a sequence.
- **Statistics** Students will analyze data using the graphing calculator.
- **Geometry** Students will work with triangles to analyze patterns.

Grade/Level:

Grades 6-8

Duration/Length:

This activity will take 2 days. The Enrichment Worksheet will take an additional day.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Recognizing patterns
- Developing formulas from tables
- Operating the TI-80, TI-81, or TI-82 calculator (scatter plots, graphing, line/curve of best fit, regression equation)

- Analyzing sequences (Students should have had adequate practice with this.)
- Recognizing linear, quadratic, and exponential functions

Objectives:

Students will:

- work cooperatively in groups.
- collect data from a geometric model.
- organize data in a table.
- develop a formula based on a geometric model and a data table.
- write about the mathematical relationships they discover.
- use the TI-80 calculator to list data, analyze a scatter plot, and make a hypothesis as to an appropriate model that best fits the data.

Materials/Resources/Printed Materials:

- Pencils
- Paper
- TI-80, TI-81, or TI-82 calculator
- Worksheets #1 & #2

Development/Procedures:

Day 1:

- Review different types of triangles.
- Review mathematical sequences and let students know that geometric figures can be a model for these sequences.
- Assign students to small groups.
- Have student groups work on Worksheet #1. (Each student should complete their own worksheet.)
- Monitor student's work as they complete the table and derive the formula. It may be necessary to interrupt and discuss their results as a class.
- Depending on student's experience with the graphing calculator, #8-10 on the worksheet may be done together as a class. If students do these questions independently in their groups, allow sufficient time for class discussion.
- Preview homework assignment.

Day 2:

- Review homework assignment (see Extension/Follow Up).
- Hand out Worksheet #2 and let students continue to work in groups.
- Allow adequate time for students to explore the pattern presented by the table. This process will require them to be patient and to use good number sense. They must be willing to initiate a trial and error process.

- When groups have develop their formulas discuss their results as a class. (They should all get the following formula: $y = [x(x + 1)] / 2$.)

- Allow students to complete #3-6 in their groups.

- Discuss results as a class. The quadratic should be the curve of best fit, but be sure they explore the linear and exponential functions as well. Question #6 should confirm their result in #2 -- the equations should be the same!

Day 3: Enrichment Worksheet (optional)

- Review solving simultaneous equations.

- Teach solving systems of three equations.

- Choose any three data points from the table from Worksheet #2 to develop the system of three equations. Since this is a quadratic function the general form of the equations will be $y = ax^2 + bx + c$. The students will be solving for a, b, and c.

- The solution of this system should match the result of #6 from Worksheet #2.

Evaluation:

The evaluation for this learning unit will be through two worksheets and one homework assignment. A formal assessment will be based on the project included. This project can be assigned after Worksheet #1 or Worksheet #2 and should be done independently by each student.

Extension/Follow Up:

- Use student's homework assignments from day 1 to generate discussion about the formulas they discovered. It is possible to have students present their work to the class. The class as a whole could plot the data from the tables to determine the correct equation from the calculator.

- Explore Pascal's Triangle and the patterns it holds.

- Explore patterns found in nature including the Fibonnacci Sequence.

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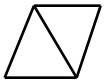
Worksheet #1: Patterns with Equilateral Triangles Name:_____

1. Sketch an equilateral triangle to the right.

2. How many sides/edges does your equilateral triangle have?_____

3. If you were to attach two triangles together (horizontally), like this 

   how many outside edges would the figure have?_____

4. Continue to attach triangles in a "horizontal manner". Use a sketch in the space below to help you complete the table given.

Number of Triangles	Number of Outside Edges
1	3
2	
3	
4	
5	

5. Describe in words how the two columns of numbers compare._____

   _____

   _____

   _____

   _____

6. Develop a formula, using the variable n, to represent the relationship between the first and second column of the table._____

7. Use your formula to determine how many outside edges there would be if there were twenty triangles attached. (Show your work.)

8. Plot the data from your table on your graphing calculator. Let x be the number of triangles and let y be the number of outside edges.

9. Do the points from the scatter plot appear to lie on a straight line or a curve? _____

   Now use your calculator to find the line of best fit. What is the equation of this line?

   _____

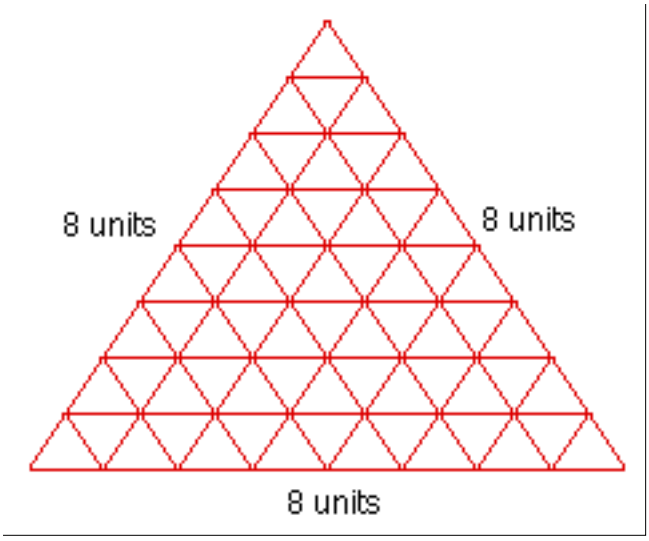
10. Use the table function on your calculator to check the answer you gave to question #7.

HOMEWORK

Follow steps 1-7 above but use a polygon other than a triangle. For instance, a pentagon has five sides. If you were to attach two pentagons together, how many outside edges would you have? You should have steps 1-7 completed for the polygon of your choice. Be sure to include a visual model.

Worksheet #2: Patterns: Vertical Equilateral triangles (△) Name:_____

Purpose: We are given an equilateral triangle of side 8 units. First, we count the total number of vertical equilateral triangles of length 1 unit that are included in this big triangle. Next, we vary the length of the equilateral triangle and again count the total number of vertical equilateral triangles of length 1 unit that make up the new triangle. Finally, we check to see if there is a pattern between the length of the equilateral triangles and the total number of vertical equilateral triangles that are included in it.



1. Use the above figure to complete the table given below. The first one has been done for you.

Length of base (units)	Total no. of vertical equilateral triangles whose base=1unit
8	36
7	
6	
5	
4	
3	
2	
1	

2. Let X be the length of the base and let Y be the total number of vertical triangles. Develop a formula, using X and Y, to represent the relationship between the first and second column of the table._____

(This trial and error process may take several attempts. Be patient and use good number sense.)

3. Use your formula to determine how many vertical equilateral triangles there would be if the length of the base equaled 15 units._____
Show your work.

4. Plot the data from your table on your graphing calculator. Again, let X be the length of the base and let Y be the total number of vertical equilateral triangles.

5. Experiment with different models for this data, including linear, quadratic, or exponential -- check each! Which model seems to fit the best?

_____

6. Take the formula you found in Step 2 and compare it to the equation in your calculator that represents the best model. How do the two compare?

_____

_____

Enrichment Worksheet Name:_____

Use the concept of a system of equations (three equations), to algebraically solve question #2 in worksheet #2.

Hint: Choose any three points from the table in question 1, worksheet #2, to establish your three equations.

Project: Data Collection and Analysis

Name:_____

Description

This project requires that you collect and record daily temperatures over eight days. You need to pick two times during the day that you will record the temperature, both in Fahrenheit and Celsius. For example, you may choose to record the temperature at 7 a.m. and at 4 p.m. each day. At the end of your data collection over the ten consecutive days, you should have a total of 16 temperatures recorded.

Procedure

1. Record the temperatures as described above in the table provided on the next page.

2. Use your calculator to graph the scatter plot of your data points.

   a. Describe your scatter plot.

_____

_____

3. Experiment with the calculator to find a line of best fit.

   a. What is the general form of the equation that represents your line of best fit?

      _____

4. Utilize the calculator to discover the equation of your fitted line.

5. Record your equation: _____

Date	Time	Celsius	Fahrenheit